

Bohr Atom

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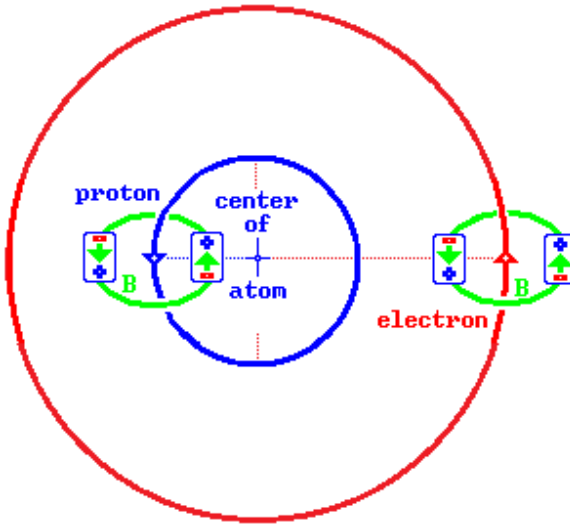


Figure 1: Bohr Atom

Abstract

Neils Bohr unified Rutherford's nuclear atom with Balmer's formula for the spectrum of the hydrogen atom in 1913 leaving us with the indelible image of the planetary atom. The planetary atom is the prototype of tiny machines which illuminate matter.

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Authors Note

This document was written with Latex <http://latex-project.org/ftp.html> and TexStudio <http://texstudio.sourceforge.net/>, both of which are excellent, open-source and free. The PDF pages it produces can be read in two page view and printed two pages at a time in landscape to save paper or make a book.

1 Neils Bohr

unified Rutherford's nuclear atom with Balmer's formula for the spectrum of the hydrogen atom in 1913 leaving us with the indelible image of the planetary atom. The planetary atom is the prototype of tiny machines which illuminate matter. Planetary systems rotate in a plane while atomic systems precess from a plane into ellipsoids which we see in scanning tunneling microscope pictures but this does not detract from Bohr's work. We address the source of this in precession [2] and electric gravity [3].

Balmer's 1885 formula, which fits many of the lines in the hydrogen spectrum can be written,

$$wavelength = \frac{4\pi r_c}{\alpha^3 base^2} \frac{n^2}{n^2 - base^2} \quad (1.1)$$

where r_c is the classical radius of the atom and α is the constant 0.00729735 or $1/137.035989$.

$$wavelength = 91.126705 \text{ nm} \frac{base^2 n^2}{n^2 - base^2} \quad (1.2)$$

for wavelengths greater than $91.1E-9 \text{ m}$.

The base=1 in the ultraviolet Lyman series and n is greater than one. When $n=2$ the Lyman series returns a wavelength of $121.5E-9 \text{ m}$, called hydrogen Lyman-alpha.

The base=2 in the visible Balmer series of spectral lines and n is greater than two. When $n=3$ the Balmer series returns a wavelength of $656.3E-9 \text{ m}$, the red of hydrogen-alpha.

2 Bohr's original atom

First - Bohr used Rydberg who used Balmer so Bohr also fits the spectral data. Bohr linked Rydberg's formula with the kinetic energy changes, due to the transition or jumping of an electron between various orbits, in electrons orbiting the nucleus of atoms. These kinetic energy changes emit and absorb photons visible in the hydrogen spectrum.

We will look first at the original almost universally seen Bohr's atom where the proton is considered the unmoving center of the atom. The fallacy of the unmoving center introduces conceptual errors which are propagated forward to thinking that the Sun is, likewise, the unmoving center of our solar system. This misses the

wobble [4] of the Sun and how inertia, angular momentum and centrifugal force work. It introduces errors in mass, velocity and distance which are corrected only when you consider the electron and proton as a binary system or the Sun and planets as a system. Everything moves in an orbital system. There is no unmoving center.

The electron and proton both orbit around a point between their centers which is called the barycenter, the center of mass or the center of the atom. The electron is at a distance of r_e from the barycenter. The proton is at a distance of r_p from the barycenter. The distance between the electron and proton is cd , the center distance. They are in a line so $cd = r_e + r_p$. Only if you say that the proton has an unmoving center and $r_p = 0$, can you say that the electron orbits the proton at a distance of $r_e = cd$. In reality, cd is always bigger than r_e . We will use $r_e = cd$ for a few more steps.

Second - Bohr postulated the angular momentum of an electron in various concentric orbits around a proton equals multiples of $\frac{\text{Plank's constant}}{2\pi}$ or $\frac{n \cdot h_p}{2\pi}$. We now know $\frac{h_p}{4\pi}$ as the spin of the electron.

The angular momentum of the atom is multiples of twice the spin of the electron, an agreeable symmetry. m_e is the mass of the electron, v_e is its tangent velocity and cd is the orbital center distance. n is the interger orbit counter moving out from $n = 1$.

The angular momentum is n units of Plank's constant h_p , divided by 2π .

$$m_e v_e cd = \frac{h_p n}{2\pi} \quad (2.1)$$

$$v_e = h_p \cdot \frac{n}{2\pi cd m_e} = \frac{2\pi r_c m_e c}{\alpha} \cdot \frac{n}{2\pi cd m_e} = \frac{n r_c c}{cd \alpha} \quad (2.2)$$

Isolated v_e , the velocity of the electron at the n orbit. Substituted for h_p . r_c is the classical radius of the electron and α is the fine structure constant.

The velocity and center distance must vary with elliptical orbits but the angular moment remains constant. When the velocity goes up the center distance goes down.

$$\uparrow \cdot \downarrow = \frac{n r_c c}{cd \alpha} \quad (2.3)$$

Third - the electron orbits so the centrifugal force equals the centripetal force. When a force exerts a center seeking centripetal

force, inertia opposes this deviation from straight line motion with a center fleeing centrifugal force. The centripetal force equals the inertial centrifugal force along a circular orbital path. For every action there is an equal but opposite reaction.

Slinging a rock on a rope, around in a circle, demonstrates this centrifugal force which can easily be measured with a spring scale used by fishermen to weigh their fish or air travelers to weigh their luggage.

You and the rock are masses in a binary system. Your centrifugal force, at your distance from the barycenter, the center of mass, equals the centrifugal force of the rock, at its distance from the common center of mass, equals the tension in the rope between the two masses. If the rope is cut or released both the centripetal and centrifugal forces become zero. The rock continues on its inertial path. You continue on your inertial path. Both paths are in opposite directions. They are determined by the momentum prior to release and are tangent to the circle at the point of release.

The centripetal Coulomb force is the electrostatic attractive force between an electron in orbit with a proton, both with a charge of ce at a separation center distance of cd meters.

$$\frac{m_e v_e^2}{cd} = \frac{ce^2}{4\pi e_0 cd^2} \dots\dots\dots \text{or} \dots\dots\dots m_e v_e^2 cd = \frac{ce^2}{4\pi e_0} \quad (2.4)$$

The electron centrifugal force equals the electron-proton Coulomb force but

$$r_c = \frac{ce^2}{4\pi e_0 m_e c^2} \dots\dots\dots \text{and} \dots\dots\dots m_e c^2 = \frac{ce^2}{4\pi e_0 r_c} \quad (2.5)$$

where r_c is the classical radius of the electron [6]. The rest energy $m_e c^2$, of the hydrogen atom is equal to the energy of the charge inside r_c . We can write: $\frac{ce^2}{4\pi e_0} = \frac{ce^2}{4\pi e_0}$.

$$m_e v_e^2 cd = m_e c^2 r_c \dots\dots\dots \text{or} \dots\dots\dots v_e^2 cd = c^2 r_c \quad (2.6)$$

$$v_e^2 = \frac{c^2 r_c}{cd} \dots\dots\dots \text{or} \dots\dots\dots cd = \frac{c^2 r_c}{v_e^2} \quad (2.7)$$

2.1 Electron velocity v_e

$$v_e^2 = \frac{c^2 r_c}{cd} \dots\dots\dots \text{from equation} \dots\dots\dots (2.7)$$

$$v_e = \frac{c r_c}{cd} n \dots \dots \dots \text{from equation } \dots \dots \dots (2.2)$$

$$\frac{v_e^2}{v_e} = v_e = \frac{c^2 r_c cd \alpha}{n c r_c cd} = \frac{c \alpha}{n} = \frac{\text{equation (2.7)}}{\text{equation (2.2)}} \quad (2.8)$$

$$v_e = \frac{c \alpha}{n} = \frac{c}{137.036 n} = 2,187,691 \frac{m}{s} \quad (2.9)$$

the velocity is a small fraction of the speed of light.

$$v_{e1}^2 = \frac{c^2 \alpha^2}{n^2} \dots \dots \dots \text{equation } \dots \dots \dots (2.10)$$

2.2 Electron radius of orbit cd

$$v_{e2}^2 = \frac{c^2 r_c}{cd} \dots \dots \dots \text{from equation } \dots \dots \dots (2.7)$$

$$v_{e2}^2 = v_{e1}^2$$

$$\frac{c^2 r_c}{cd} = \frac{c^2 \alpha^2}{n^2} \dots \text{or } \dots \frac{r_c}{cd} = \frac{\alpha^2}{n^2} \dots \text{or } \dots cd \alpha^2 = r_c n^2$$

$$cd = \frac{r_c n^2}{\alpha^2} = 5.292E-11 m n^2 \quad (2.11)$$

The center distance between the electron and proton is cd. cd is $1/\alpha = 137.036$ times larger than the ring electron. The ring electron has a radius of r_c/α and the first Bohr orbit a radius of r_c/α^2 , an interesting symmetry.

2.3 Electron angular momentum

$$m_e v_e cd = \frac{n h_p}{2\pi} \dots \dots \dots \text{from equation } \dots \dots \dots (2.1)$$

substitute $v_e = c \alpha/n$ and $cd = r_c n^2/\alpha^2$.

$$m_e \frac{c \alpha}{n} \frac{r_c n^2}{\alpha^2} = \frac{n h_p}{2\pi} \quad (2.12)$$

$$\frac{m_e c r_c}{\alpha} = \frac{h_p}{2\pi} \dots \dots \dots \text{or } \dots \dots \dots h_p = \frac{2\pi m_e c r_c}{\alpha} \quad (2.13)$$

This is twice the spin of the electron. Isolated h_p as a check.

The energy of the photon in the atom is said to equal the KE_e , kinetic energy difference of the electron alone as it jumps between various orbits. We will see that since the atom also includes the proton that any transition in the atom must include the transition of the proton when looking at the electron-proton atom as a binary system.

2.4 Electron kinetic energy

$$KE_e = h_p \text{ frequency} = .5 m_e v_e^2 = .5 m_e \frac{c^2 \alpha^2}{n^2} \quad (2.14)$$

substituted for ve^2 .

$$KE_e = J = 2.17987E-18 \frac{kg \ m^2}{s^2 \ n^2} = 13.6057 \frac{eV}{n^2} \quad (2.15)$$

Substitute for h_p and frequency.

$$h_p \text{ frequency} = \frac{2\pi r_c m_e c}{\alpha} \frac{c}{\text{wavelength}} = .5 m_e \frac{c^2 \alpha^2}{n^2} \quad (2.16)$$

$$\text{wavelength} = \frac{4\pi r_c n^2}{\alpha^3} = 91.126705E-9 \ m \ n^2 \quad (2.17)$$

$$\frac{1}{\text{wavelength}} = \frac{\alpha^3}{4\pi r_c n^2} = 10.973E6 \ \frac{1}{m} \ \frac{1}{n^2} \quad (2.18)$$

This is called the Rydberg constant. A 13.605698 eV binding energy photon has a wavelength of 91.126705E-9 m, 91 nano meters.

2.5 Photon kinetic energy

We are working with differences in kinetic energy in electron volts or Joules and as a last step converting to wavelength. If the base=2 and n=4 then we can write the kinetic energy of the photon as the difference of the kinetic energy of the base and the kinetic energy of n. A higher n or larger orbit has less energy.

$$\frac{KE_e}{base^2} - \frac{KE_e}{n^2} = KE \text{ of the photon} \quad (2.19)$$

$$\frac{13.6057 \ eV}{2^2} - \frac{13.6057 \ eV}{4^2} = 2.551065 \ eV \quad (2.20)$$

$$2.551065 \text{ eV} \cdot 1.602177E-19 \frac{J}{eV} = J = \quad (2.21)$$

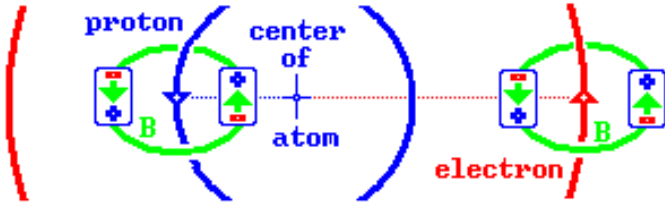
$$4.087256E-19 \frac{kg \text{ m}^2}{s^2} \frac{1}{h_p c} = \frac{1}{486.0097E-9 \text{ m}} \quad (2.22)$$

Working with all these units written out and conversions between various units is so much easier if you use a HP-48 thru HP-50g graphics calculators, not the newer HP-Prime, which is not recommended. The older HP calculators are emulated quite well on Android phones.

Sommerfeld added a correction for elliptical orbits and the relativistic increase in mass with velocity as the electron moves from apogee to perigee. This causes the orbit to precess and to trace rosettes in the same way that Mercury precesses in its orbital plane. Atoms must also precess out of their orbital plane for us to see atoms as spherical in scanning tunneling microscope pictures.

Physics seems to be drifting away from a physical reality. Do you think we could have arrived at Rydberg's constant, by the above circuitous route, without a substantial portion of Bohr's planetary atom being correct?

3 Bohr's atom as a binary system



On figure (3), a binary system generalizes Bohr's planetary atom with an unmoving center proton to a system where both the electron and proton move. The Sun also orbits or wobbles around the center of mass of the solar system, the barycenter. This figure shows red-electron and blue-proton currents and green magnetic fields which are forces due to moving charge [17].

The binary systems contains the proper amount of energy to agree with the Balmer series hydrogen spectral lines while also agreeing with the energy of ionization.

When the hydrogen atom is ionized the electron-proton pair separate and absorb energy. It takes the binding energy to pull them apart. A laser might knock an electron loose.

When a electron-proton pair merge to become a hydrogen atom they give off this energy.

3.1 Orbiting charge

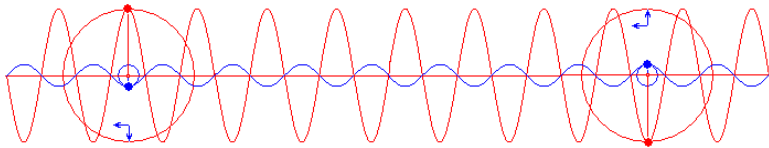


Figure 2: Orbiting charge

On figure (2), the large red sine waves are electrons and the small blue sine waves are protons orbiting on their separate helical paths. The sine waves are an edge view of the helical orbital path and currents traced out by the electron and proton pair as they move across the page on a path like a spring on a string.

Both orbit each other on opposite sides of the center of mass of the system.

There are both wave and particle descriptions of atoms. We will be looking at both.

The orbital period of the proton and electron pair is the same. They are a dipole.

Looking at a point, in the orbital plane as they orbit, would show alternating charges and dipole forces at the frequency that the electron and proton orbit and pass in front of each other. Plus-minus-plus-minus at $6.6E15$ hertz as the dipoles reverse direction at a wavelength of $45.5E-9$ m in the extreme ultraviolet.

A distant static charge would only see the oscillating high frequency plus-minus-plus-minus merged to neutrality which excludes gravity from being caused by the interaction of dipoles and static charges.

A distant in-phase rotating dipole would however experience Coulomb, magnetic and gravitational forces.

The stationary unmoving center proton and orbiting electron view of the planetary atom conceals their binary wavelike behavior which the sine waves emphasize. Matrix mechanics [5] is equivalent to wave equations for future reference.

3.2 Binary systems

r_e and r_p are the distances for the electron and proton to the center of mass, the barycenter, of the electron-proton system. m_e and m_p are their masses. v_e and v_p are their orbital velocities.

$$m_e v_e = m_p v_p \quad (3.1)$$

The momentum of the electron and proton are equal.

$$m_e r_e = m_p r_p \quad (3.2)$$

The mass times distance products are equal and balanced.

$$\frac{v_e}{r_e} = \frac{v_p}{r_p} \quad (3.3)$$

This is equation (3.1) divided by equation (3.2). The angular velocity and orbital periods are equal.

$$\frac{m_e v_e^2}{r_e} = \frac{m_p v_p^2}{r_p} \quad (3.4)$$

This is equations (3.1) times (3.3) the centrifugal forces are equal to each other.

$$cd = r_e + r_p \quad (3.5)$$

where cd is the center distance between the electron and proton.

$$\begin{aligned} cd &= r_e + m_e r_e / m_p, \text{ since } r_p = m_e r_e / m_p \\ cd &= r_e (1 + m_e / m_p) = r_e (m_p + m_e) / m_p \\ 1/k &= m_p + m_e / m_p = 1.0005446 = 1 + 1/1836.15 \\ cd &= r_e / k = r_e (1 + 1/1836.15) = r_e \cdot 1.0005446 \\ cd &= m_p r_p / m_e + r_p, \text{ since } r_e = m_p r_p / m_e \\ cd &= r_p (m_p / m_e + 1) = r_p (m_p + m_e) / m_e \end{aligned}$$

$$cd = \frac{r_c n^2}{\alpha^2} = 5.29178E-11 \text{ m } n^2 \quad (3.6)$$

but cd must vary in an elliptical orbit.

$$r_e = \frac{cd}{k} n^2 = \frac{r_c}{\alpha^2} \frac{m_p}{m_p + m_e} n^2 = 5.28889E-11 \text{ m } n^2 \quad (3.7)$$

the distance between the electron and the center of mass.

$$rp = cd \frac{m_e n^2}{m_p + m_e} = \frac{r_c}{\alpha^2} \frac{m_e n^2}{m_p + m_e} = 2.88042E-14 \text{ m } n^2 \quad (3.8)$$

the distance between the proton and the center of mass for orbit n .

$$v_e = \frac{c \alpha}{n} = \frac{c}{137.036 n} = 2,187,691.56 \frac{m}{s n} \quad (3.9)$$

the velocity for orbit n of the electron around the center of mass.

$$v_p = v_e \frac{m_e}{m_p} = \frac{c \alpha}{n} \frac{m_e}{m_p} =$$

$$\frac{m_e}{m_p} \frac{c}{137.036 n} = 1,191.45 \frac{m}{s n} = 2,665.2 \text{ miles per hour} \quad (3.10)$$

the velocity, for orbit $n=1$, of the proton around the center of mass.

$$\text{frequency} = \frac{v_e}{2\pi r_e} = \frac{c \alpha}{2\pi r_e} = 6.58327E15 \frac{1}{s} = \quad (3.11)$$

6.6 peta hertz.

$$\text{wavelength} = \frac{c}{\text{frequency}} = \frac{2\pi r_e}{\alpha} = 45.54E-9 \text{ m}. \quad (3.12)$$

This is in the extreme ultraviolet, EUV. Here is a reference to the Solar Dynamics Observatory, [8].

“EUV wavelengths range between 50 and 5 nanometers, which coincide with the characteristic absorption wavelengths of inner-shell electrons in the atoms that compose matter. As a result, EUV light directed onto a standard mirror or lens at normal incidence is absorbed rather than reflected, making it undetectable. For this reason, EUV light is also absorbed by Earth’s atmosphere, which is why telescopes must travel to space to study the light emitted from the Sun.”

3.3 Two different angular momentums

We have two different angular momentums, one for the electron and one for the proton:

$$m_e v_e r_e + m_p v_p r_p = \frac{h_p}{2\pi} n$$

the sum of the electron and proton angular momentum in increments of Planck’s constant divided by 2π .

Substitute $m_e v_e = m_p v_p$.

$$m_e v_e r_e + m_e v_e r_p = \frac{h_p}{2\pi} n$$

$$m_e v_e (r_e + r_p) = m_e \frac{c \alpha k}{n} \frac{r_c n^2}{k \alpha^2} = \frac{h_p n}{2\pi} \quad (3.13)$$

$cd = r_e + r_p$. When value of v_e goes up then the value of cd must go down for the right hand side of the equation, $h_p/2\pi$, to remain unchanged. That is what the following k does.

Substitute for $v_e = \frac{c \alpha k}{n}$ and $cd = \frac{r_c}{k \alpha^2}$.

$$m_e \frac{c \alpha k}{n} \frac{r_c n^2}{k \alpha^2} = \frac{h_p}{2\pi} n \quad (3.14)$$

the k 's cancel in this equation. The k 's will be used to adjust the value of two other equations below while keeping the overall value of the present equation unchanged.

$$\frac{m_e c r_c}{\alpha} = \frac{h_p}{2\pi} \dots\dots \text{and} \dots\dots h_p = \frac{2\pi r_c m_e c}{\alpha} \quad (3.15)$$

collected terms. This includes one spin for the electron and one spin for the proton.

Centrifugal force of the electron equals the centrifugal force of the proton equals the Coulomb force

$$\frac{m_e v_e^2}{r_e} = \frac{m_p v_p^2}{r_p} = \frac{ce^2}{4\pi e_0 cd^2} = 8.2297E^{-8} \frac{kg m}{s^2} \quad (3.16)$$

$$v_e = c \alpha k$$

$$cd = r_c / (\alpha^2 k)$$

$$r_e = cd k = r_c / \alpha^2$$

These are fantastically strong forces with respect to the tiny masses. The electron and proton are locked together. The electron acceleration,

$$\frac{force}{mass} = \frac{m_e v_e^2}{m_e r_e} = \frac{v_e^2}{r_e} = 9.034E22 \frac{m}{s^2}. \quad (3.17)$$

The Earth's $g = 9.8 m/s^2$ is a hundred billion trillion times smaller.

We can isolate k as follows:

$$\frac{m_e v_e^2}{r_e} = \frac{ce^2}{4\pi e_0 cd^2} \quad (3.18)$$

the centrifugal force equals the Coulomb force.

$$\frac{m_e c^2 \alpha^2 k^2}{r_e} = \frac{m_e c^2 r_c}{cd^2} \quad (3.19)$$

where $m_e c^2 r_c = ce^2/(4\pi e_0)$ and $r_e = cd m_p/(m_e + m_p)$

$$\frac{\alpha^2 k^2 (m_e + m_p)}{cd m_p} = \frac{r_c}{cd^2} \quad (3.20)$$

$$\frac{\alpha^2 k^2 (m_e + m_p)}{m_p} = \frac{r_c}{cd} \quad (3.21)$$

where $cd = r_c/(k \alpha^2)$

$$\frac{\alpha^2 k^2 (m_e + m_p)}{m_p} = r_c \frac{k \alpha^2}{r_c} \quad (3.22)$$

$$k = \frac{m_p}{(m_e + m_p)} = 0.999455679421 = 1 - 0.000544320578 \quad (3.23)$$

$$v_e = c \alpha k = c \alpha \frac{m_p}{m_e + m_p} \quad (3.24)$$

$$cd = \frac{r_c}{\alpha^2 k} = \frac{r_c}{\alpha^2} \frac{m_e + m_p}{m_p} \quad (3.25)$$

We know the electron and proton pair can have elliptical paths as this is required for atoms to be polarized and precess into polarized ellipsoids which can attract each other. See electric gravity [3].

The energy of the photons in the atom equals the kinetic energy of the electron and the proton. We will see that since the atom also includes the proton that any transition in the atom must include the transition of the proton. There is a division of the energy between the binding energy of the proton and electron.

3.4 Kinetic energy for the electron

electron kinetic energy = $KE_e =$

$$h_p \cdot frequency = .5 m_e v_e^2 = .5 m_e c^2 \alpha^2 \frac{k^2}{n^2}$$

$$\frac{2\pi r_c m_e c}{\alpha} \frac{c}{\text{wavelength}} = .5 m_e c^2 \alpha^2 \frac{k^2}{n^2} = J$$

substituted for h_p and frequency.

$$\frac{2\pi r_c m_e c}{\alpha} \frac{c}{\text{wavelength}} = .5 m_e c^2 \alpha^2 \frac{m_p^2}{(m_e + m_p)^2 n^2} \quad (3.26)$$

$$\frac{1}{\text{wavelength}} = \frac{\alpha^3 k^2}{4\pi r_c n^2} = J \quad (3.27)$$

$$\frac{1}{\text{wavelength}} = \frac{\alpha^3 m_p^2}{4\pi r_c (m_e + m_p)^2 n^2} = 10.96178E6 \frac{1}{m} \frac{1}{n^2} \quad (3.28)$$

This is the Rydberg constant for the electron less the proton.

$$2.1775E-18 \frac{kg m^2}{s^2 n^2} = 13.59089 \frac{eV}{n^2} \quad (3.29)$$

The binding energy for the electron.

$$\text{wavelength} = \frac{4\pi r_c (m_e + m_p)^2 n^2}{\alpha^3 m_p^2} = 91.226E-9 m n^2 \quad (3.30)$$

91.2 nano meters.

3.5 Kinetic energy for the proton

proton kinetic energy = $KE_p =$

$$h_p \text{ frequency} = .5 m_p v_p^2 = .5 m_p c^2 \alpha^2 \frac{m_e^2}{(m_e + m_p)^2} \quad (3.31)$$

$$h_p \text{ frequency} = .5 m_p \frac{v_e^2 m_e^2}{m_p^2} = .5 m_p c^2 \alpha^2 \frac{m_p^2}{(m_e + m_p)^2 n^2} \frac{m_e^2}{m_p^2}$$

$$\text{and where } v_e^2 = \frac{c^2 \alpha^2 k^2}{n^2} = c^2 \alpha^2 \frac{m_p^2}{(m_e + m_p)^2 n^2}$$

$$\frac{2\pi r_c m_e c}{\alpha} \frac{c}{\text{wavelength}} = .5 m_p \frac{m_e^2 c^2 \alpha^2}{(m_e + m_p)^2} = J$$

substituted for h_p and frequency.

$$\frac{1}{\text{wavelength}} = \frac{\alpha^3 m_e m_p}{4\pi r_c (m_e + m_p)^2 n^2} = 5969.97626 \frac{1}{m} \frac{1}{n^2} \quad (3.32)$$

This is the Rydberg constant for a proton.

$$1.1859E-21 \frac{kg \ m^2}{s^2 \ n^2} = 0.0074018 \frac{eV}{n^2} \quad (3.33)$$

The binding energy of the proton.

$$wavelength = \frac{4\pi r_c (m_e + m_p)^2 n^2}{\alpha^3 m_e m_p} = 1.675049E-4 \ m \ n^2 \quad (3.34)$$

These proton transitions produce photons in the range of microwave or infrared waves. When the electrons produce light the protons produce heat. The electron and proton both do something like a Hohmann transfer to change orbits. Has anyone noticed these long wavelength proton transitions which seem to be required for ionization?

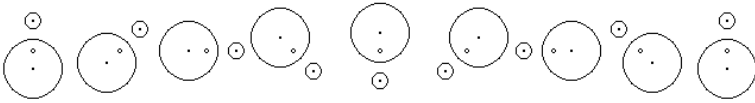


Figure 3: Binary orbits, both masses always move

4 Balmer series hydrogen spectral lines

$$wavelength = \frac{4\pi r_c}{\alpha^3} \frac{base^2 n^2}{n^2 - base^2} \quad (4.1)$$

$$wavelength = 91.126705 \ nm \frac{base^2 n^2}{n^2 - base^2} \quad (4.2)$$

$$\frac{1}{wavelength} = 10.973E6 \ \frac{1}{m} \left(\frac{1}{base^2} - \frac{1}{n^2} \right) \quad (4.3)$$

Wavelength table in nano-meters

n	base	measured	Bohr	Bohr (1+(me/mp))
3	2	656.2852 nm	656.1128 nm	656.4702 nm
3	2	656.2720 nm	656.1128 nm	656.4702 nm
4	2	486.1330 nm	486.0095 nm	486.2742 nm
5	2	434.0470 nm	433.9372 nm	434.1734 nm
6	2	410.1740 nm	410.0705 nm	410.2938 nm
7	2	397.0072 nm	396.9078 nm	397.1239 nm
8	2	388.9049 nm	388.8076 nm	389.0193 nm
9	2	383.5384 nm	383.4426 nm	383.6514 nm

First three columns; n, base, measured wavelength from hyper-physics [14].

Fourth column, calculated wavelength from Bohr, Balmer or Rydberg.

$$wavelength = 91.126705 \text{ nm} \frac{base^2 n^2}{n^2 - base^2} \quad (4.4)$$

these wavelengths are a little too short so the transitions show too much energy.

Fifth column, calculated wavelength from Bohr (1+me/mp) considering the hydrogen atom as a binary system.

$$wavelength = 91.176410 \text{ nm} \frac{base^2 n^2}{n^2 - base^2} \quad (4.5)$$

these wavelengths are a little too long so the transitions show not quite enough energy. The proton also has long wavelength and low energy transitions which occur at the same time as the electron transitions but they are not listed.

The first two rows:

3 2 656.2852 nm = 1.889182 eV, n=3, base=2, hydrogen alpha, electron spin up?

3 2 656.2720 nm = 1.889220 eV, n=3, base=2, hydrogen alpha, electron spin down? The measured difference in the wavelength 0.0132 nm or 0.000038 eV between these first two rows is attributed to the spin of the proton being spin up and the spin of the electron being either spin up or spin down. See the forces due to moving charge [17].

The third row - using the 486.133 nm spectral line:

$$486.133E-9 \text{ m} = 486.133 \text{ nm} = 4.086E-19 \frac{\text{kg m}^2}{\text{s}^2} = 2.55 \text{ eV}$$

which is the correct value for the 4 to 2 transition line in the Balmer series in the hydrogen spectrum. Here

$keE = .5 m_e v_e^2 = .5 m_e c^2 \alpha^2 k^2 = 13.602 \text{ eV}$. The k in v_e of the kinetic energy of the electron ke_e , is adjusted to give the correct kinetic energy of the electron to yield the correct wavelength of the photon.

$$ke_e/4 - ke_e/16 = 4.086E-19 \text{ kg m}^2/\text{s}^2 = 2.55 \text{ eV}$$

$$k = 0.999872969911 = 1 - 0.000127030089,$$

so the kinetic energy changes by about one part in ten thousand to yield the exact wavelength.

$ke_e = 13.6022303559 \text{ eV}$ is a $91.1499E-9 \text{ m}$ photon or a slightly revised Rydberg constant of 10970934 1/m .

The third row:

4 2 2.550418 eV 2.551066 eV 2.549878 eV, n = 4, base = 2, shown

in eV. We see the difference in the electron volt equivalents is about 0.0006 eV. This is easily hidden in the room temperature thermal noise of about 0.04 eV, as in the following calculation.

$$.5 \text{ mass velocity}^2 = 3/2 k_B T = 0.037892 \text{ eV},$$

the kinetic energy caused by thermal velocity at

$$T = 20 \text{ C} = 68 \text{ F} = 293 \text{ K}.$$

$$\text{Boltzmann's constant} = k_B = 8.617385E-5 \text{ eV/K}.$$

See hyperphysics [14].

5 Ionization

is the energy added to separate the electron and proton in a hydrogen atom. It can also be seen as the binding energy given off when the electron and proton become a hydrogen atom. It is the sum of the kinetic energy of the electron and proton.

NIST gives 13.5984 eV for the ionization energy.

5.1 electron kinetic energy

$$\text{electron kinetic energy} = ke_e = .5 m_e v_e^2 \quad (5.1)$$

$$\text{where } v_e^2 = c^2 \alpha^2 k^2 = c^2 \alpha^2 \frac{m_p^2}{(m_e + m_p)^2}$$

$$ke_e = .5 m_e \frac{c^2 \alpha^2 m_p^2}{(m_e + m_p)^2 n^2} = J = \quad (5.2)$$

$$2.17749984211E-18 \frac{\text{kg m}^2}{\text{s}^2 \text{ n}^2} = 13.5908791202 \frac{\text{eV}}{\text{n}^2} \quad (5.3)$$

5.2 proton kinetic energy

$$\text{proton kinetic energy} = ke_p = .5 m_p v_p^2 \quad (5.4)$$

$$\text{where } v_p = v_e \frac{m_e}{m_p} = \frac{c \alpha m_p}{m_e + m_p} \frac{m_e}{m_p} = \frac{c \alpha m_e}{m_e + m_p}$$

$$ke_p = .5 m_p \frac{c^2 \alpha^2 m_e^2}{(m_e + m_p)^2 n^2} = J =$$

$$1.18590347957E-21 \frac{\text{kg m}^2}{\text{s}^2 \text{ n}^2} = J = 0.00740182411375 \frac{\text{eV}}{\text{n}^2} \quad (5.5)$$

The proton kinetic energy is close to $\alpha \frac{eV}{n^2}$.

The electron kinetic energy plus the proton kinetic energy =

$$\begin{aligned} & 13.5909 \text{ eV} \\ & + .0074 \text{ eV} \\ & = 13.5983 \text{ eV} \end{aligned}$$

The sum of the electron and proton energy closely agrees with the NIST value of 13.5984 eV.

This is the total kinetic energy of the electron-proton pair in the hydrogen atom. When the Bohr atom is considered as a binary system with a moving proton, it gives the NIST value. This kinetic energy is able to remove both the electron and proton to infinity. It is also said to be the combined binding energy of both the electron and proton into an atom of hydrogen.

The energy in the electrostatic field equals the sum of the kinetic energies:

$ce^2/(4\pi e_0 cd)$, this is the energy in the electrostatic field for the electron-proton pair, substitute

$m_e c^2 r_c = ce^2/(4\pi e_0)$ and

$cd = r_e + r_p = r_c(m_e + m_p)/(m_p \alpha^2)$ so,

$$\frac{m_e c^2 r_c m_p \alpha^2}{r_c(m_e + m_p)} = \frac{m_e m_p c^2 \alpha^2}{m_e + m_p} \quad (5.6)$$

collected terms, the binding energy is half the energy of the electrostatic field so,

$$.5 \frac{m_e m_p c^2 \alpha^2}{m_e + m_p} = 2.1786857 E^{-18} \frac{kg \ m^2}{s^2} = 13.59828 \text{ eV} \quad (5.7)$$

equals the sum of the electron and proton kinetic energies. This is the hydrogen atom mass deficit since ionization separates the electron and proton.

We previously calculated, 13.6022 eV yields the exact spectral lines. Here we see 13.5908 eV for the electron or 0.0114 eV difference or that of hydrogen heated from absolute zero Kelvin to 132 Kelvin = -221 F. Apparently we need to know the temperature at which the lines in the hydrogen spectrum are measured to resolve this small difference in the energy of the spectral lines.

Mass deficits: See Wiki binding energy.

Binding energy [10] or NIST index

NIST constants [11] or NIST constants

Physics constants [12]

1 amu = 1/12 of the mass of a C_{12} atom =

1.66053886E-27 kg/amu

From NIST [9] for the hydrogen atom mass for comparison with our calculation below.

$13.5984 \text{ eV}/c^2 = 2.42\text{E-}35 \text{ kg}$ is small when compared to the mass of the hydrogen atom.

Proton = 1.007276 amu

Electron = 0.000549 amu

Proton plus an electron = 1.00782499918 amu

Hydrogen atom = 1.00782498458 amu,

The mass of ionization = 0.00000001459 amu = $2.42\text{E-}35 \text{ kg}$

The mass of ionization for the electron alone is $13.5910 \text{ eV}/c^2$

The mass of ionization for the proton alone is $0.0074 \text{ eV}/c^2$

The mass of ionization for the hydrogen atom is $13.5984 \text{ eV}/c^2$

6 The Virial theorem

says the kinetic energy equals half the gravitational potential energy.

$$.5 m v_t^2 = \frac{.5 G m M}{r} \quad (6.1)$$

Multiply by $2/r$.

$$\frac{m v_t^2}{r} = \frac{G m M}{r^2} \quad (6.2)$$

The centrifugal force equals the gravitational force. Collect terms.

$$v_t^2 r = G M \quad (6.3)$$

It works just as well with electrostatic forces.

7 Binding energy

The energy required to leave an orbit and escape to infinity is called the Binding energy [16]. Both the potential and kinetic energy at infinity are zero. The kinetic energy is always positive. Gravitational energy is taken as negative.

$-.5 G m M/r$, is the energy in orbit so, the energy to make the kinetic energy zero at infinity is, $.5 G m M/r$, the energy added so, $.5 G m M/r$, is the binding energy.

8 Reference

This is a wonderful book, /Hydrogen The Essential Element/ by John H. Rigden.

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