Reaction Wheels

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January 6, 2016

Abstract

This paper looks at Reaction Wheels, like those used in space craft. It was copied from an excellent paper from 1959 [1]. The poor quality PDF was re-written in Latex with the hope of making them more legible and usable.

Key Words

Reaction Wheels

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Authors Note

This document was written with Latex http://latex-project.org/ftp.html and TexStudio http://texstudio.sourceforge.net/, both of which are excellent, open-source and free. The PDF pages it produces can be read in two page view and printed two pages at a time in landscape to save paper or make a book.

1 Appendix

A. Disturbance Sources
Disturbance torques which act to perturb the vehicle from the desired orientation may arise from the following sources:
1) Rotating parts within the vehicle.
2) Inertial cross-coupling due to differences in principal moments of inertia.
3) Interaction with ambient gravitational. magnetic, and electric fields.
4) Incident and emitted radiation.
5) Particle impingements.
6) Aerodynamics.
Because of present lack of knowledge of the environment to which a space vehicle will be subjected, only crude estimates of the magnitudes of disturbances can be made. It is difficult to generalize since these disturbances depend upon the vehicle configuration. The vehicle should, then, be designed to minimize the disturbing effects.
B. Sensing Methods
There are a number of sensing methods that can be incorporated into a space vehicle control system. These methods can be divided into four basic classes:
1) Sensing by inertial means. Devices which fall into this category are gyroscopes, accelerometers, pendulums, vibrating masses. etc.
2) Sensing by sighting of celestial bodies. Such devices are horizon scanners, sun seekers, moon seekers, and star trackers.
3) Sensing by interaction with ambient fields. Such methods are quite restricted, in that ambient fields vary with altitude and orbital position. For very low altitudes use can be made of the atmospheric pressure gradient. The earth'a magnetic field varies considerably with orbital position, so that some programming must be used with such a system.

The observation of the differential gravitational forces on the various parts of the vehicle requires extremely sensitive instrumentation in measuring this gravitational gradient. 4) Sensing by ground observation of signals transmitted from the vehicle. The field pattern of a narrow radar beam transmitted from the vehicle can yield attitude information. Correction signals are then transmitted to the vehicle from the ground station. Although the sensing methods listed above have their limitations and disadvantages, the control designer has a relatively broad selection of feasible metA.vls of attitude sensing.

2 Equations of Motion

Let the reference coordinate system consist of the xyz axes, with the origin iocated at the center of mass of the vehicle. It is assumed, for generality, that the position of the vehicle center of mass is known as a function of time. The angular velocity $\overrightarrow{\omega}_{ref}$ of the reference frame, then, is also known as a function of time. It has components $\omega_x, \ \omega_y, \ and \ \omega_z$ directed along the x, y, and z axes, respectively.

$$\overrightarrow{\omega}_{ref} = \omega_x \overrightarrow{e}_x + \omega_y \overrightarrow{e}_y + \omega_z \overrightarrow{e}_z \tag{2.1}$$

Let XYZ form an orthogonal set of body-fixed axes directed along the principal axes of inertia of the vehicle, which has principal moments of inertia $I_X, \ I_Y, \ and \ I_Z$. The angular velocity Cbody of these body-fixed axes in inertial space has components $\omega_X, \ \omega_Y, \ and \ \omega_Z$ directed along the X, Y, and Z axes, respectively.

$$\overrightarrow{\omega}_{body} = \omega_X \overrightarrow{e}_X + \omega_Y \overrightarrow{e}_Y + \omega_Z \overrightarrow{e}_Z \tag{2.2}$$

Since the sensible, elements will, in general, ,measure deviations from the reference, it may be desirable to express the body rates in terms of these deviations. The angular velocity of the reference frame, the body axes, and the angular velocity $\overrightarrow{\omega}_{rel}$ of the body axes relative to the reference frame, are related by:

$$\overrightarrow{\omega}_{body} = \overrightarrow{\omega}_{ref} + \overrightarrow{\omega}_{rel} \tag{2.3}$$

Let us define these deviation angles by the three rotations, $\theta_3, \ \theta_1, \ and \ \theta_2$ in that order, which transform the reference axes, xyz, into the body axes, XYZ. θ_3 is a rotation about the z axis, transforming xyz into x'y'z.
θ_1 is a rotation about the x' axis, transforming x'y'z into x'Yz'. θ_2 is a rotation about the Y axes, transforming x'Yz' into XYZ.

This sequence was selected so that $\theta_1 \ and \ \theta_2$ correspond to the gimbal angles of a gimballed "pointer" that is aligned with one of the reference axes. The choice of deviation angles, however, is arbitrary. The unit vectors in the two sets of axes are then related by:

$$\begin{pmatrix} e_X \\ e_Y \\ e_Z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix}$$

where

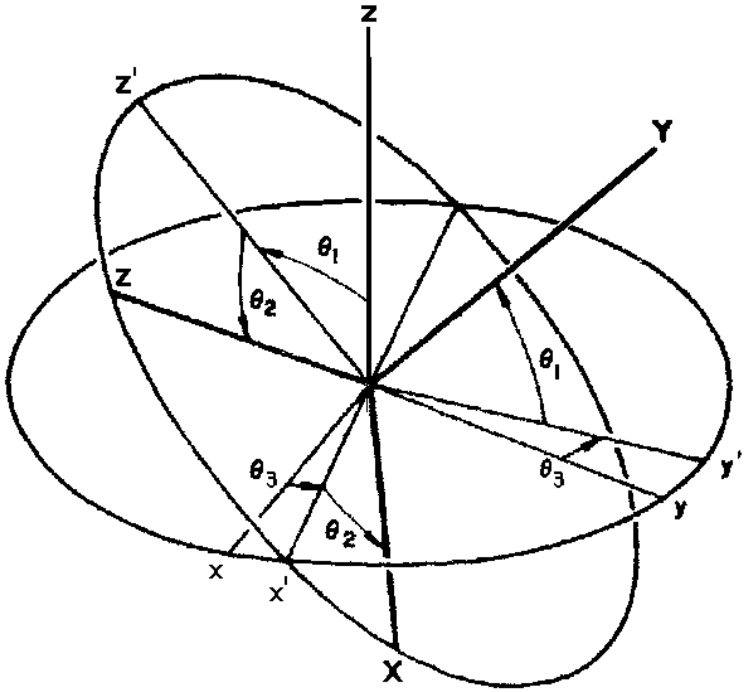

Figure 1: Deviation Angles

$$
\begin{aligned}
a_{11} &= cos_2 cos_3 - sin\theta_1 sin\theta_2 sin\theta_3 \\
a_{12} &= cos_2 sin_3 + sin\theta_1 sin\theta_2 sin\theta_3 \\
a_{13} &= \neg cos_1 sin_2 \\
a_{21} &= \neg cos_1 sin_3 \\
a_{22} &= cos_1 cos_3 \\
a_{23} &= \neg cos_1 sin_2 \\
a_{31} &= sin_2 cos_3 + sin\theta_1 cos\theta_2 sin\theta_3 \\
a_{32} &= sin_2 sin_3 + sin\theta_1 cos\theta_2 cos\theta_3 \\
a_{33} &= cos_1 cos_2
\end{aligned} \tag{2.4}
$$

This transformation is shown in diagram figure (1)

Euler's equations, which describe the general motion of a body about its center of mass, are:

$$
\begin{aligned}
I_X\dot{\omega}_X + (I_Z - I_Y)\omega_Z\omega_Y &= (L_D)_X + (L_\omega)_X \\
I_Y\dot{\omega}_Y + (I_X - I_Z)\omega_X\omega_Z &= (L_D)_Y + (L_\omega)_Y \\
I_Z\dot{\omega}_Z + (I_Y - I_X)\omega_Y\omega_X &= (L_D)_Z + (L_\omega)_Z \qquad (2.5)
\end{aligned}
$$

where the terms on the right-hand side of equation (2.5) are the components of the external disturbance torque (T_D) on the vehicle and (T_ω) the torque due to the motion of the wheels.

Using equations (2.3) and (4.4), the components of the vehicle angular velocity in inertial space directed along the body axes, then become:

$$
\begin{aligned}
\omega_X &= a_{11}\omega_x + a_{12}\omega_y + a_{13}(\omega_z + \dot{\theta}_3)) + \dot{\theta}_1 cos\theta_2 \\
\omega_Y &= a_{21}\omega_x + a_{22}\omega_y + a_{23}(\omega_z + \dot{\theta}_3) + \dot{\theta}_2 \\
\omega_Z &= a_{31}\omega_x + a_{32}\omega_y + a_{33}(\omega_z + \dot{\theta}_3)) + \dot{\theta}_1 sin\theta_2 \qquad (2.6)
\end{aligned}
$$

Let $H_X,\ H_Y\ and\ H_Z$ represent the components of the total angular momentum, H, of the X, Y, and Z wheels relative to the vehicle frame. The torque upon the vehicle due to the motion of the wheels becomes:

$$\overrightarrow{L_\omega} = \neg\frac{d}{dt}(H_X\overrightarrow{e_X} + H_Y\overrightarrow{e_Y} + H_Z\overrightarrow{e_Z}) \qquad (2.7)$$

or

$$\neg\overrightarrow{L_\omega} = \dot{H}_X\overrightarrow{e_X} + \dot{H}_X\overrightarrow{e_X} + \dot{H}_Y\overrightarrow{e_Y} \qquad (2.8)$$

The components of L_ω along the body axes are:

$$
\begin{aligned}
\neg(\overrightarrow{L_\omega})_X &= \dot{H}_X + H_Z\omega_Y - H_Y\omega_Z \\
\neg(\overrightarrow{L_\omega})_Y &= \dot{H}_Y + H_X\omega_Z - H_Z\omega_X \\
\neg(\overrightarrow{L_\omega})_Z &= \dot{H}_Z + H_Y\omega_X - H_X\omega_Y \qquad (2.9)
\end{aligned}
$$

The general equations of motion of a vehicle with reaction wheels, then, are given by equations (2.5), (2.6) and (2.9). The moments of inertia of the vehicle are now defined to include the moments of inertia of the wheels. These equations are not in a form that can be suitably used for conventional control system synthesis. However, they can be reduced to a more tractable form by linearization, so that the resulting expressions are ordinary differential equations with constant coefficients, Conventional servo techniques can then be applied.

For the single-axis laboratory model the equation of motion, excluding disturbances, is

$$I\ddot{\theta} = \neg\dot{H} = \neg J\dot{\omega} \tag{2.10}$$

where
I = platform momert of inertia
θ = platform angular deviation
H = wheel momentum
J = wheel moment of inertia
ω = wheel speed
The control equation is

$$\dot{H} = \mu K_m J \frac{(s+k)(t_1 s+1)}{(t_m s+1)(t_2 s+1)}(\theta_C - \theta) \tag{2.11}$$

where
μ =amplifier gain
K_m = motor gain
k = integrator gain
t_m = motor time constant
t_1 = lead time constant of shaping network
t_2 = lag time constant of shaping network
θ_C = commanded platform deviation
Amplifier and integrator gains and lead-lag time constants were selected to yield a 50 per cent damped system.

References

[1] Ronald W. Froelich and Harry Patapoff *REACTION WHEEL ATTITUDE CONTROL FOR SPACE VEHICLES* Presented at the IRE National Automatic Control Conference November 6, 1959 Dallas, Texas http://www.dtic.mil/dtic/tr/fulltext/u2/608027.pdf

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